Multiple Access Regardless of Time- and Frequency-Selective Fading

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Abstract

Relying on block spreading and despreading, where the spreading and despreading matrices corresponding to the same user only depend on this user's signature code, we propose a novel multi-user transceiver that achieves deterministic multi-user separation in the presence of unknown time- and frequency-selective fading. More important, Maximum Likelihood (ML) optimality is preserved in this multi-user separation step in the presence of additive white Gaussian noise. In addition to MUI-resilient reception, channel-irrespective symbol detectability is also guaranteed. The maximum achievable spectral efficiency is analyzed, and an implementation based on conventional symbol spreading and despreading combined with an (I)FFT operation and interleaving is presented. Simulation results show improved performance of the proposed transceiver over a multi-user time-frequency RAKE receiver for Direct-Sequence (DS) Code Division Multiple Access (CDMA).

1 Introduction

Relying on block spreading and despreading, where the spreading and despreading matrices corresponding to the same user only depend on this user's signature code, various multi-user transceivers have recently been proposed to achieve deterministic multi-user separation in the presence of unknown time-flat frequency-selective fading, and guarantee channel-irrespective symbol detectability [2], [10], [12], [4]. However, all these multi-user transceivers assume that the underlying channels are time-invariant over the transmitted frame. In practice, this assumption does not hold true when channel time-variation arises due to high-mobility, carrier frequency offsets, and phase drifts.

In this paper, we deal with multiple access in the presence of doubly-selective (time- and frequency-selective) channels, and rely on a basis expansion channel model, which has previously been studied in [1], [7], [9]. Starting from this model, we then propose a multi-user transceiver, relying on block spreading and despreading, where the spreading and despreading matrices corresponding to the same user only depend on this user's signature code. It turns out that mutual orthogonality among users is preserved even after unknown time- and frequency-selective propagation. As a result, deterministic multi-user separation is achieved in the presence of unknown time- and frequency-selective fading. More important, ML optimality is preserved in this multi-user separation step in the presence of additive white Gaussian noise. Hence, a multi-user detection problem is converted into a set of *equivalent* single-user equalization problems. In addition to MUI-resilient reception, channel-irrespective symbol detectability is also guaranteed. The proposed MUI-resilient transceiver serves as an alternative to multi-user time-frequency RAKE receivers for DS-CDMA [8], which do not guarantee channel-irrespective symbol detectability, and require the knowledge of the spreading codes of all active users. For a doubly-selective channel, the proposed transceiver achieves a maximum spectral efficiency of about $(1-\sqrt{S})^2$, where S is the multi-user channel spread factor (c.f. [3]). Hence, for highly underspread multi-user channels ($S \ll 1$), the proposed transceiver can achieve bandwidth efficient multiple access. Finally, the proposed transceiver can be implemented based on conventional symbol spreading and despreading combined with an (I)FFT operation and interleaving. This is particularly useful at the base station, where computation and storage resources can be shared by all users.

Notations: We denote the Kronecker delta as $\delta[\cdot]$ and the Kronecker product as $\otimes$. The $M \times N$ all-zero matrix is denoted as $\mathbf{0}_{M\times N}$, the $N \times N$ identity matrix as $\mathbf{I}_N$, and the $N \times N$ unitary Fast Fourier Transform (FFT) matrix as $\mathbf{F}_N$. Furthermore, $\mathbf{\Lambda}_N(\nu)$ is the $N \times N$ diagonal matrix with main diagonal $[1, e^{j2\pi\nu}, \dots, e^{j2\pi(N-1)\nu}]^T$. We define $\mathbf{i}_N[n]$ as the $(n+1)$st column of $\mathbf{I}_N$, and $\mathbf{f}_N[n]$ as the $(n+1)$st column of $\mathbf{F}_N$. We denote $[\mathbf{A}]_{m,n}$ as the $(m+1, n+1)$st entry of the matrix $\mathbf{A}$. Finally, $\mathbb{Z}_0^+$ ($\mathbb{R}_0^+$) represents the set of non-negative integer (real) numbers, whereas $\mathbb{Z}^+$ ($\mathbb{R}^+$) represents the set of positive integer (real) numbers.

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2 Basis expansion model

Suppose the uth user's ($u \in \{0,\ldots,U-1\}$) symbol sequence $s_u[m]$ is somehow converted into a chip sequence $x_u[n]$ that is to be transmitted with chip period T_c. Let us define $g_u(t;\tau)$ as the causal doubly-selective continuous time channel for the uth user, and let us assume that the maximal time-offset of $g_u(t;\tau)$ is bounded by τ_{max} and the maximal frequency-offset of $g_u(t;\tau)$ is bounded by f_{max}. These assumptions can easily be satisfied in practice [5]. After chip rate sampling, the received sequence $y[n]$ can then be written as

$$y[n] = \sum_{u=0}^{U-1} \sum_{\nu=-\infty}^{\infty} g_u[n;\nu] x_u[n-\nu] + \eta[n],$$

where $\eta[n]$ is additive noise and $g_u[n;\nu] := g_u(nT_c;\nu T_c)$. If we now design a triplet (N,L,Q), with $N \in \mathbb{Z}^+$ and $L,Q \in \mathbb{Z}_0^+$, that satisfies condition C1): $L \geq \lfloor \tau_{\text{max}}/T_c \rfloor$, and condition C2): $Q/N \geq f_{\text{max}} T_c$, the channel $g[n;\nu]$ can very accurately be modeled by a discrete-time channel [5]

$$h_u[n;\nu] = \sum_{l=0}^{L} \delta[\nu - l] \sum_{q=-Q}^{Q} h_{u,l,q}[\lfloor n/N \rfloor] e^{j2\pi qn/N}.$$

Note that similar basis expansion models have been investigated in [1], [7], [9].

3 Proposed transceiver

The proposed transceiver starts from the design of a triplet (N,L,Q) that satisfies conditions C1) and C2), such that $g_u[n;\nu]$ can be modeled by $h_u[n;\nu]$, and uses the chosen triplet (N,L,Q) in the receiver design as well as in the transmitter design.

We use block spreading to convert the uth user's symbol stream $s_u[m]$ into a chip stream $x_u[n]$, which is done as follows. The symbol sequence $s_u[m]$ is first serial-to-parallel converted into a sequence of $M \times 1$ symbol blocks $\mathbf{s}_u[i] = [s_u[iM],\ldots,s_u[(i+1)M-1]]^T$. Each symbol block $\mathbf{s}_u[i]$ is then spread by an $N \times M$ spreading matrix $\mathbf{C}_u$ to obtain a sequence of $N \times 1$ chip blocks $\mathbf{x}_u[i] = \mathbf{C}_u \mathbf{s}_u[i]$. This sequence of chip blocks $\mathbf{x}_u[i]$ is finally parallel-to-serial converted into a chip stream $x_u[n] = \mathbf{i}_N^T[n \bmod N] \mathbf{x}_u[\lfloor n/N \rfloor]$. With chip rate sampling, the received sequence $y[n]$ can then be written as

$$y[n] = \sum_{u=0}^{U-1} \sum_{l=0}^{L} h_u[n;l] x_u[n-l] + \eta[n],$$

as discussed in the previous section.

At the receiver, we serial-to-parallel convert the received sequence $y[n]$ into a sequence of received blocks $\mathbf{y}[i] := [y[iN],\ldots,y[(i+1)N-1]]^T$, which can be written as

$$\mathbf{y}[i] = \sum_{u=0}^{U-1} \sum_{q=-Q}^{Q} \mathbf{\Lambda}_N\left(\tfrac{q}{N}\right) \cdot \sum_{a=0}^{\lceil L/N \rceil} \mathbf{H}_{N,u,q}^{(a)}[i] \mathbf{C}_u \mathbf{s}_u[i-a] + \boldsymbol{\eta}[i], \quad (1)$$

where $\boldsymbol{\eta}[i] := [\eta[iN],\ldots,\eta[(i+1)N-1]]^T$, and $\mathbf{H}_{N,u,q}^{(a)}[i]$ represents the $N \times N$ Toeplitz matrix with $[\mathbf{H}_{N,u,q}^{(a)}[i]]_{n,n'} = h_{u,n-n'+aN,q}[i]$. Note that $\lceil L/N \rceil + 1$ symbol blocks $\mathbf{s}_u[i]$ contribute to each received block $\mathbf{y}[i]$; the terms with $a > 1$ represent the so-called Inter Block Interference (IBI).

To extract the uth user, we despread $\mathbf{y}[i]$ by a despreading matrix $\mathbf{D}_u$ to obtain

$$\bar{\mathbf{y}}_u[i] = \mathbf{D}_u^H \mathbf{y}[i]. \quad (2)$$

On the MUI-resilient output $\bar{\mathbf{y}}_u[i]$, we can then apply any single-user equalizer to mitigate the Inter Symbol Interference (ISI). We can, e.g., adopt a linear single-user equalizer $\mathbf{\Gamma}_u[i]$ to obtain $\hat{\mathbf{s}}_u[i] = \mathbf{\Gamma}_u[i] \bar{\mathbf{y}}_u[i]$.

Next, we will design transceiver pairs $\{(\mathbf{C}_u,\mathbf{D}_u)\}_{u=0}^{U-1}$ that achieve deterministic multi-user separation in the presence of unknown time- and frequency-selective channels, and guarantee channel-irrespective symbol detectability.

3.1 Basic results

We design a quintuplet (N,L,Q,K,P), with $N,K,P \in \mathbb{Z}^+$ and $L,Q \in \mathbb{Z}_0^+$, that satisfies conditions C1), C2), and condition C3): $N = U(P+2Q)(K+L)$. As symbol block length, we then take $M = PK$. Such a quintuplet (N,L,Q,K,P) will be referred to as an admissible quintuplet. In the remainder of this section, we will assume that there always exists an admissible quintuplet (N,L,Q,K,P).

To each user u, we assign a distinct $U \times 1$ signature vector $\mathbf{c}_u = [c_u[0],\ldots,c_u[U-1]]^T$. The code vectors $\{\mathbf{c}_u\}_{u=0}^{U-1}$ are selected to be mutually orthogonal, i.e., $\mathbf{c}_u^H \mathbf{c}_{u'} = \delta[u-u']$. Let the $(K+L) \times K$ zero-inserting matrix be defined as $\mathbf{T}_1 = [\mathbf{I}_K, \mathbf{0}_{K \times L}]^T$, and the $(P+2Q) \times P$ zero-inserting matrix as $\mathbf{T}_2 = [\mathbf{0}_{P \times Q}, \mathbf{I}_P, \mathbf{0}_{P \times Q}]^T$. Relying on the FFT matrix $\mathbf{F}_{U(P+2Q)}$, we then design the spreading matrix $\mathbf{C}_u$ as the $N \times M$ matrix given by

$$\mathbf{C}_u = [\mathbf{F}_{U(P+2Q)}^H (\mathbf{c}_u \otimes \mathbf{T}_2)] \otimes \mathbf{T}_1, \quad (3)$$

and the despreading matrix $\mathbf{D}_u$ as the $N \times (P+2Q)(K+L)$ matrix given by

$$\mathbf{D}_u = [\mathbf{F}_{U(P+2Q)}^H (\mathbf{c}_u \otimes \mathbf{I}_{P+2Q})] \otimes \mathbf{I}_{K+L}. \quad (4)$$

Note that $\mathbf{C}_u$ and $\mathbf{D}_u$ only depend on the uth user's code vector $\mathbf{c}_u$ and a common FFT matrix $\mathbf{F}_{U(P+2Q)}$. Starting from the orthogonal code vectors $\{\mathbf{c}_u\}_{u=0}^{U-1}$, we first observe that $\mathbf{C}_u$ and $\mathbf{D}_u$ possess mutual orthogonality among users:

$$\begin{aligned} \mathbf{C}_u^H \mathbf{C}_{u'} &= \delta[u-u'] \mathbf{I}_M, \\ \mathbf{D}_u^H \mathbf{D}_{u'} &= \delta[u-u'] \mathbf{I}_{(K+L)(P+2Q)}. \end{aligned} \quad (5)$$

Choosing $\mathbf{c}_u = \mathbf{i}_U[u]$ leads to an FDMA-like transmission, whereas choosing $\mathbf{c}_u = \mathbf{f}_U[u]$ leads to a TDMA-like transmission [5]. Other sets of orthogonal code vectors, e.g., the set of Walsh-Hadamard code vectors, lead to CDMA-like transmissions, which have the potential to be more robust against narrow band noise/interference and burst noise/interference.

For *time-flat frequency-selective channels*, we can set $Q = 0$ and $P = 1$, such that $\mathbf{C}_u = (\mathbf{F}_U^H \mathbf{c}_u) \otimes \mathbf{T}_1$ and $\mathbf{D}_u = (\mathbf{F}_U^H \mathbf{c}_u) \otimes \mathbf{I}_{K+L}$. This transceiver scheme corresponds to the Chip Interleaved Block Spread (CIBS) CDMA transceiver scheme of [12], if we use $\{\mathbf{F}_U^H \mathbf{c}_u\}_{u=0}^{U-1}$ as code vectors in [12].

For *time-selective frequency-flat channels*, we can set $L = 0$ and $K = 1$, such that $\mathbf{C}_u = \mathbf{F}_{U(P+2Q)}^H(\mathbf{c}_u \otimes \mathbf{T}_2)$ and $\mathbf{D}_u = \mathbf{F}_{U(P+2Q)}^H(\mathbf{c}_u \otimes \mathbf{I}_{P+2Q})$. If we then take $\mathbf{c}_u = \mathbf{i}_U[u]$, we obtain $\mathbf{C}_u = \mathbf{F}_{U(P+2Q)}^H(\mathbf{i}_U[u] \otimes \mathbf{T}_2)$ and $\mathbf{D}_u = \mathbf{F}_{U(P+2Q)}^H(\mathbf{i}_U[u] \otimes \mathbf{I}_{P+2Q})$. This transceiver scheme corresponds to the uplink transceiver scheme of [6], if we ignore the cyclic prefix in [6]. This cyclic prefix was actually introduced in [6] to enable the extension from frequency-flat to frequency-selective channels. However, although this extension allows deterministic multi-user separation in the presence of unknown time- and frequency-selective channels, it does not guarantee channel-irrespective symbol detectability. The proposed transceiver, on the other hand, achieves both, as we will explain further on.

One immediate consequence of the design of $\mathbf{C}_u$ in (3) is that IBI is removed, since we have that $\lceil L/N \rceil = 1$ and $\mathbf{H}_{N,u,q}^{(1)}[i]\mathbf{C}_u = \mathbf{0}_{N\times M}$ in (1). Hence, we only need to perform block by block processing on the following IBI-free blocks:

$$\mathbf{y}[i] = \sum_{u=0}^{U-1} \sum_{q=-Q}^{Q} \mathbf{\Lambda}_N(\tfrac{q}{N})\mathbf{H}_{N,u,q}^{(0)}[i]\mathbf{C}_u\mathbf{s}_u[i] + \boldsymbol{\eta}[i]. \tag{6}$$

Note that although $\mathbf{C}_u$ possesses mutual orthogonality among users [c.f. (5)], the equivalent spreading matrix seen at the receiver: $\bar{\mathbf{C}}_u := \sum_{q=-Q}^{Q} \mathbf{\Lambda}_N(\frac{q}{N})\mathbf{H}_{N,u,q}^{(0)}[i]\mathbf{C}_u$ does not retain this orthogonality, in general, after unknown time- and frequency-selective propagation. However, we next show that the judiciously designed $\mathbf{C}_u$ in (3) guarantees that such an orthogonality is preserved.

Let us introduce the notation $\mathbf{J}_{N,q}^{(a)}$, to represent the $N \times N$ Toeplitz matrix with $[\mathbf{J}_{N,q}^{(a)}]_{n,n'} = \delta[n - n' - q + aN]$, and define the matrices $\mathbf{H}_{u,q}[i] := \mathbf{H}_{K+L,u,q}^{(0)}[i]\mathbf{T}_1$, $\mathbf{\Delta}_q := \mathbf{\Lambda}_{K+L}(\frac{q}{N})$, and $\mathbf{J}_q := \mathbf{J}_{P+2Q,q}^{(0)}\mathbf{T}_2$. We can then show that (see [5] for a proof):

$$\mathbf{H}_{N,u,q}^{(0)}[i]\mathbf{C}_u = \mathbf{A}_u(\mathbf{I}_P \otimes \mathbf{H}_{u,q}[i]),$$
$$\mathbf{\Lambda}_N(\tfrac{q}{N})\mathbf{A}_u = \mathbf{D}_u(\mathbf{J}_q \otimes \mathbf{\Delta}_q),$$

where $\mathbf{A}_u = [\mathbf{F}_{U(P+2Q)}^H(\mathbf{c}_u \otimes \mathbf{T}_2)] \otimes \mathbf{I}_{K+L}$. From the above equalities, we then obtain

$$\begin{aligned}\bar{\mathbf{C}}_u &= \textstyle\sum_{q=-Q}^{Q} \mathbf{\Lambda}_N(\tfrac{q}{N})\mathbf{H}_{N,u,q}^{(0)}[i]\mathbf{C}_u \\ &= \mathbf{D}_u \textstyle\sum_{q=-Q}^{Q} (\mathbf{J}_q \otimes \mathbf{\Delta}_q)(\mathbf{I}_P \otimes \mathbf{H}_{u,q}[i]) \\ &= \mathbf{D}_u \textstyle\sum_{q=-Q}^{Q} \mathbf{J}_q \otimes (\mathbf{\Delta}_q\mathbf{H}_{u,q}[i]) := \mathbf{D}_u\boldsymbol{\mathcal{H}}_u[i],\end{aligned} \tag{7}$$

where $\boldsymbol{\mathcal{H}}_u[i] := \sum_{q=-Q}^{Q} \mathbf{J}_q \otimes (\mathbf{\Delta}_q\mathbf{H}_{u,q}[i])$ has the following form:

$$\boldsymbol{\mathcal{H}}_u[i] = \begin{bmatrix} \mathbf{\Delta}_{-Q}\mathbf{H}_{u,-Q}[i] & & \\ \vdots & \ddots & \\ \mathbf{\Delta}_{Q}\mathbf{H}_{u,Q}[i] & & \mathbf{\Delta}_{-Q}\mathbf{H}_{u,-Q}[i] \\ & \ddots & \vdots \\ & & \mathbf{\Delta}_{Q}\mathbf{H}_{u,Q}[i] \end{bmatrix}. \tag{8}$$

From (7), it is clear that $\bar{\mathbf{C}}_u$ falls in the column space of $\mathbf{D}_u$. Since $\mathbf{D}_u$ possesses mutual orthogonality among users [c.f. (5)], we conclude that *mutual orthogonality among users is preserved after unknown time- and frequency-selective propagation*, i.e.,

$$\begin{aligned}\mathbf{D}_u^H\bar{\mathbf{C}}_{u'} &= \mathbf{D}_u^H \textstyle\sum_{q=-Q}^{Q} \mathbf{\Lambda}_N(\tfrac{q}{N})\mathbf{H}_{N,u',q}^{(0)}[i]\mathbf{C}_{u'} \\ &= \delta[u-u']\boldsymbol{\mathcal{H}}_u[i].\end{aligned} \tag{9}$$

Using (9) and (6), we can then express (2) as

$$\bar{\mathbf{y}}_u[i] = \mathbf{D}_u^H\mathbf{y}[i] = \boldsymbol{\mathcal{H}}_u[i]\mathbf{s}_u[i] + \bar{\boldsymbol{\eta}}_u[i], \tag{10}$$

where $\bar{\boldsymbol{\eta}}_u[i] := \mathbf{D}_u^H\boldsymbol{\eta}[i]$. As a result, deterministic multi-user separation is achieved in the presence of unknown time- and frequency-selective channels. More important, since the matrix $\mathbf{D} := [\mathbf{D}_0, \ldots, \mathbf{D}_{U-1}]$ is an $N \times N$ unitary matrix, ML optimality is preserved in this multi-user separation step in the presence of additive white Gaussian noise, as is the case in [12] for time-flat frequency-selective channels. Hence, a multi-user detection problem has been converted into a set of *equivalent* single-user equalization problems. On the other hand, since the non-zero matrices from the set $\{\mathbf{\Delta}_q\mathbf{H}_{u,q}[i]\}_{q=-Q}^{Q}$ always have full column rank, and the set $\{\mathbf{\Delta}_q\mathbf{H}_{u,q}[i]\}_{q=-Q}^{Q}$ always contains at least one non-zero matrix, $\boldsymbol{\mathcal{H}}_u[i]$ always has full column rank. Therefore, the symbols are guaranteed to be detectable in the absence of noise. As a result, channel-irrespective symbol detectability is guaranteed. This is in contrast with conventional multi-user systems, e.g., DS-CDMA systems, where signals from different users may cancel each other for some bad channels. Finally, based on [11], it is possible to show that our design in (3) also enables maximum multipath-Doppler diversity.

On the MUI-resilient output $\bar{\mathbf{y}}_u[i]$, we can then apply any single-user equalizer to mitigate the ISI. We can, e.g., adopt the linear ZF equalizer given by

$$\mathbf{\Gamma}_u^{\text{zf}}[i] := (\boldsymbol{\mathcal{H}}_u^H[i]\mathbf{R}_{u,\bar{\eta}}^{-1}\boldsymbol{\mathcal{H}}_u[i])^{-1}\boldsymbol{\mathcal{H}}_u^H[i]\mathbf{R}_{u,\bar{\eta}}^{-1},$$

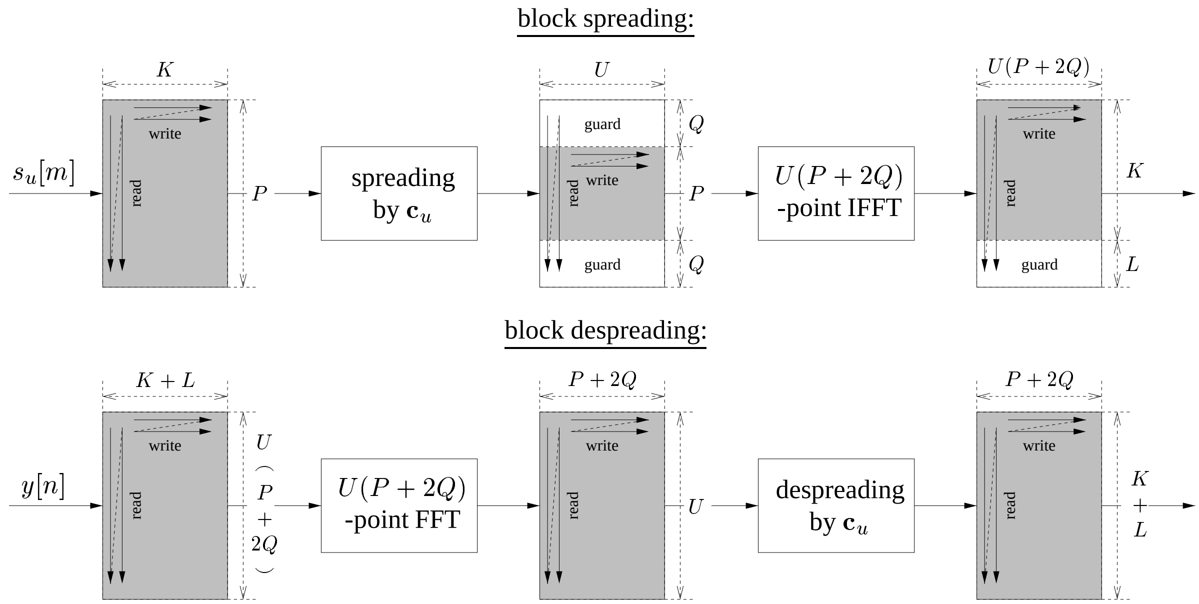

Figure 1. Implementation based on interleaving.

or the linear MMSE equalizer given by

$$\mathbf{\Gamma}_u^{\text{mmse}}[i] := (\boldsymbol{\mathcal{H}}_u^H[i]\mathbf{R}_{u,\bar{\eta}}^{-1}\boldsymbol{\mathcal{H}}_u[i] + \mathbf{R}_{u,s}^{-1})^{-1}\boldsymbol{\mathcal{H}}_u^H[i]\mathbf{R}_{u,\bar{\eta}}^{-1},$$

where $\mathbf{R}_{u,s} := \mathrm{E}\{\mathbf{s}_u[i]\mathbf{s}_u^H[i]\}$ and $\mathbf{R}_{u,\bar{\eta}} := \mathrm{E}\{\bar{\boldsymbol{\eta}}_u[i]\bar{\boldsymbol{\eta}}_u^H[i]\}$.

3.2 Spectral efficiency

Within each $N \times 1$ received block $\mathbf{y}[i]$, M symbols are transmitted per user. Hence, the spectral efficiency can be expressed as $\mathcal{E} = UM/N = PK/[(P+2Q)(K+L)]$. By tuning the parameters, we can change the spectral efficiency. However, keep in mind that the quintuplet (N, L, Q, K, P) has to be admissible. Next, we will present how to design an admissible quintuplet (N, L, Q, K, P) that reaches a spectral efficiency close (or equal) to the maximum spectral efficiency that can be reached over the set of admissible quintuplets (N, L, Q, K, P).

Using condition C3), $\mathcal{E}$ can be rewritten as $\mathcal{E} = K/(K+L) - 2U(Q/N)K$ [5]. We then observe that $\mathcal{E}$ is independent of P. It is further clear that in order to maximize $\mathcal{E}$, we should take L and Q/N as small as possible. Hence, we should take $L = L_{\text{min}} = \lfloor \tau_{\text{max}}/T_c \rfloor$, and $Q/N = (Q/N)_{\text{min}} = f_{\text{max}}T_c$. To design the optimal K we have to consider four different cases:

Case 1 ($L_{\text{min}} = 0$ and $(Q/N)_{\text{min}} = 0$): The maximum $\mathcal{E}$ over $K \in \mathbb{Z}^+$ is reached at any $K \in \mathbb{Z}^+$, i.e., $K_{\text{opt}} \in \mathbb{Z}^+$. The corresponding spectral efficiency is $\mathcal{E}_{\text{max}} = 1$.

Case 2 ($L_{\text{min}} \neq 0$ and $(Q/N)_{\text{min}} = 0$): The maximum $\mathcal{E}$ over $K \in \mathbb{Z}^+$ is reached at $K_{\text{opt}} \to \infty$. The corresponding spectral efficiency is $\mathcal{E}_{\text{max}} \to 1$.

Case 3 ($L_{\text{min}} = 0$ and $(Q/N)_{\text{min}} \neq 0$): The maximum $\mathcal{E}$ over $K \in \mathbb{Z}^+$ is reached at $K_{\text{opt}} = 1$. The corresponding spectral efficiency is $\mathcal{E}_{\text{max}} = 1 - 2U(Q/N)_{\text{min}} = 1 - 2Uf_{\text{max}}T_c$.

Case 4 ($L_{\text{min}} \neq 0$ and $(Q/N)_{\text{min}} \neq 0$): The maximum $\mathcal{E}$ over $K \in \mathbb{R}^+$ is reached at $K_{\text{opt,real}} = -L_{\text{min}} + L_{\text{min}}^{1/2}/[2U(Q/N)_{\text{min}}]^{1/2}$. However, since we want $K \in \mathbb{Z}^+$, we take $K_{\text{opt}} = \lceil -L_{\text{min}} + L_{\text{min}}^{1/2}/[2U(Q/N)_{\text{min}}]^{1/2} \rceil$. The corresponding spectral efficiency is $\mathcal{E}_{\text{max}} \approx \{1 - [2UL_{\text{min}}(Q/N)_{\text{min}}]^{1/2}\}^2 \approx [1 - (2U\tau_{\text{max}}f_{\text{max}})^{1/2}]^2$, where the first approximation becomes an equality if $-L_{\text{min}} + L_{\text{min}}^{1/2}/[2U(Q/N)_{\text{min}}]^{1/2} \in \mathbb{Z}^+$. Defining $S := 2U\tau_{\text{max}}f_{\text{max}}$ as the multi-user channel spread factor (c.f. [3]), it is clear that the more underspread the multi-user channel, the higher $\mathcal{E}_{\text{max}}$.

Hence, an admissible quintuplet (N, L, Q, K, P), for which $L = L_{\text{min}}$, $Q/N = (Q/N)_{\text{min}}$, and $K = K_{\text{opt}}$, reaches a spectral efficiency close (or equal, in Cases 1, 2, and 3; and also in Case 4, if $-L_{\text{min}} + L_{\text{min}}^{1/2}/[2U(Q/N)_{\text{min}}]^{1/2} \in \mathbb{Z}^+$) to the maximum spectral efficiency that can be reached over the set of admissible quintuplets (N, L, Q, K, P).

3.3 Implementation based on interleaving

The block spreading by $\mathbf{C}_u$ and the block despreading by $\mathbf{D}_u$ can be implemented based on conventional symbol spreading and despreading combined with an (I)FFT operation and interleaving, as depicted in Fig. 1. More details can be found in [5]. This implementation is particularly

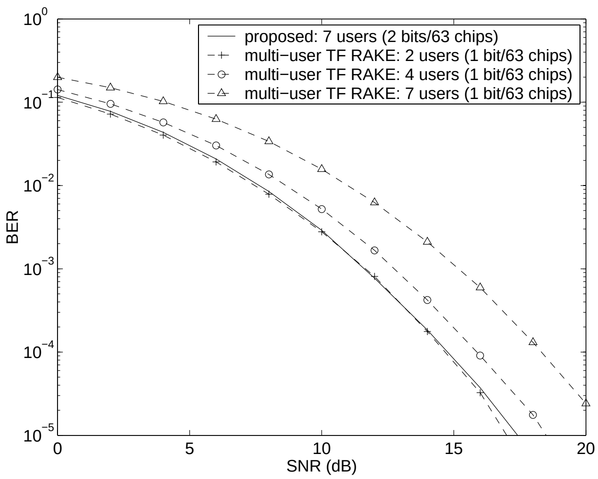

Figure 2. Proposed transceiver vs. multi-user time-frequency RAKE receiver for DS-CDMA.

useful at the base station, where the (I)FFT operation can be shared among all users to save computations, and the two redundant interleavers at the transmitter and the first two non-redundant interleavers at the receiver in Fig. 1 can be shared among all users to save storage units. Note that the non-redundant interleaver at the transmitter and the last non-redundant interleaver at the receiver are not really necessary for the multi-user separation since they just reorder the symbol block $\mathbf{s}_u[i]$ and the MUI-resilient output $\bar{\mathbf{y}}_u[i]$, respectively.

4 Simulation results

We consider an uplink scenario with BPSK modulation and additive white Gaussian noise. We focus on the same fading scenario as in [8], and compare the proposed transceiver with the multi-user time-frequency RAKE receiver for DS-CDMA [8]. As in [8], we take $N = 63$, $L = 1$, $Q = 1$, and generate a set of independent channels $\{h_u[n;\nu]\}_{u=0}^{U-1}$ with $h_{u,l,q}[i]$ complex Gaussian distributed with variance 0.9 if $q = 0$ and 0.05 if $q = \pm 1$. We assume here that the receiver knows the channels $\{h_u[n;\nu]\}_{u=0}^{U-1}$. For the proposed transceiver, N corresponds to the length of the transmitted chip block, while for the multi-user time-frequency RAKE receiver for DS-CDMA, N corresponds to the spreading factor. In order to satisfy condition C3), the proposed transceiver uses $U = 7$, $P = 1$, and $K = 2$. Note that in $N = 63$ chip periods, the proposed transceiver can handle 2 bits per user ($M = PK = 2$), while the multi-user time-frequency RAKE receiver for DS-CDMA can only handle 1 bit per user. Keeping this important rate difference in mind, Fig. 2 depicts a comparison between the performance of the proposed transceiver with linear ZF equalization and the performance of the linear ZF multi-user time-frequency RAKE receiver for DS-CDMA (the latter performance corresponds to the one shown in [8, Fig. 3]). From Fig. 2, we observe that the performance of the proposed transceiver accommodating 7 users (that transmit 2 bits/63 chips) is comparable to the performance of the multi-user time-frequency RAKE receiver for DS-CDMA accommodating only 2 users (that transmit 1 bit/63 chips), and much better than the performance of the multi-user time-frequency RAKE receiver for DS-CDMA accommodating 7 users (that transmit 1 bit/63 chips). More specifically, we gain about 4 dB for a BER of 10^{-3}. For more realistic fading scenarios we refer the interested reader to [5].

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