

COMPARISON OF DIGITAL MULTI-CARRIER WITH DIRECT SEQUENCE SPREAD SPECTRUM IN THE PRESENCE OF MULTIPATH

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ABSTRACT

We compare single user digital Multi-Carrier Spread Spectrum modulation with Direct Sequence Spread Spectrum in the presence of frequency-selective multipath fading. We derive closed-form expressions for the bit error probability and show that MC-SS is more robust to multipath fading than is DS-SS.

1. INTRODUCTION

The increasing interest in and applications of direct sequence spread spectrum (DS-SS) technology stem from its robustness to fading, its anti-interference capability, and the potential for (even uncoordinated) multiple access. With a wide bandwidth and thus a short chip period, multiple paths can be resolved with DS-SS transmissions and a RAKE receiver can be used to mitigate fading and improve system performance [6].

An alternative approach to combat frequency-selective multipath is multicarrier modulation. Multi-Carrier Spread-Spectrum (MC-SS) [7] and the corresponding multiple access scheme: Multicarrier (MC) CDMA [10] has gained increasing popularity in recent years. By exploiting multiple carriers and a narrow band DS waveform on each subcarrier, it has been shown that multicarrier DS CDMA outperforms single carrier CDMA for wideband transmissions in the presence of narrow band interference [4].

Although most existing MC approaches rely on analog carrier modulations, digital implementations through FFTs are also available [1]. Thanks to the rapid development of digital devices and digital signal processing (DSP) technologies, the Digital to Analog (D/A) and Analog to Digital (A/D) converters are being pushed closer to the transceiver's end. Starting from a discrete-time equivalent model, we investigate the performance of digital MC-SS and compare it with DS-SS. The main contributions of this paper are the novel results on performance analysis of digital MC-SS in the presence of multipath. Further results on the performance analysis of digital MC-SS in the presence of narrow band interference (NBI) and the presence of both NBI and multipath may be found in [11].

2. UNIFYING TRANSCEIVER MODELS

The diagram in the upper part of Fig. 1 describes the discrete-time baseband equivalent model of an MC-SS system. The length- N symbol periodic digital spreading code $\mathbf{c}_{mc} := [c_{mc}(0), \dots, c_{mc}(N-1)]^T$ spreads the i th information symbol $s(i)$. The resulting sequence $\mathbf{c}_{mc}s(i)$ is then IFFT processed to obtain the $N \times 1$ vector $\mathbf{F}_N^H \mathbf{c}_{mc}s(i)$, where \mathbf{F}_N is the $N \times N$ FFT matrix with (m, n) entry $(1/\sqrt{N})e^{-j2\pi mn/N}$ and H denotes Hermitian transpose.

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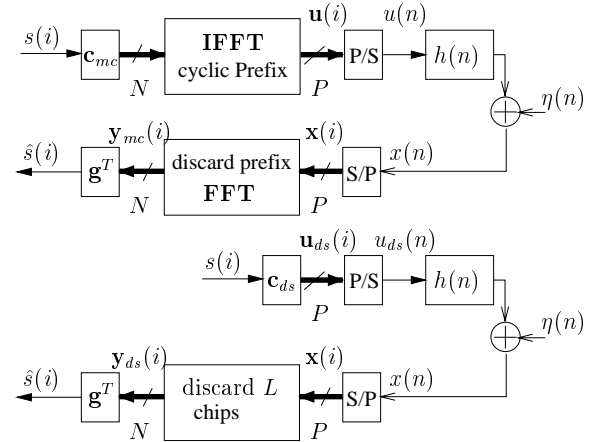


Fig. 1. Equivalent model: MC-SS (upper) and DS-SS (lower)

To avoid channel-induced inter symbol/block interference (ISI/IBI), we replicate the last $P - N$ entries (Cyclic Prefix (CP)) of the vector $\mathbf{F}_N^H \mathbf{c}_{mc}s(i)$ at the front to form the $P \times 1$ transmitted block $\mathbf{u}(i)$, as in conventional OFDM systems, e.g., [1]. The received signal, after conversion to baseband and receive filtering, is sampled at the chip rate, to yield

$$x(n) = \sum_{l=0}^L h(l)u(n-l) + \eta(n), \quad (1)$$

where $h(\ell)$ is the overall channel (transmit and receive filters, and propagation channel), $\eta(n)$ is the filtered additive Gaussian noise (AGN), and L is the maximum order of the FIR channel. To avoid ISI, the CP length should be larger than the channel order: $P - N \geq L$. To avoid bandwidth overexpansion, we choose the smallest block length $P = N + L$ here.

To convert (1) from a serial to a convenient matrix-vector form, we define the $P \times 1$ vector: $\mathbf{x}(i) := [x(iP), x(iP+1), \dots, x(iP+P-1)]^T$ (likewise for $\eta(i)$), and the $P \times P$ Toeplitz channel matrices $\mathbf{H}_0, \mathbf{H}_1$ with (k, l) th entries $h(k-l)$ and $h(k-l+P)$, respectively. Since $h(l) = 0, \forall l \notin [0, L]$, and $P = N + L$, we can write (1) as:

$$\mathbf{x}(i) = \mathbf{H}_0 \mathbf{u}(i) + \mathbf{H}_1 \mathbf{u}(i-1) + \boldsymbol{\eta}(i), \quad (2)$$

where the second term represents IBI.

At the receiver, the CP is removed by dropping the first $P - N$ elements of $\mathbf{x}(i)$, thus eliminating IBI. After the CP removal and FFT processing, we have

$$\mathbf{y}_{mc}(i) = \mathbf{F}_N \tilde{\mathbf{H}} \mathbf{F}_N^H \mathbf{c}_{mc}s(i) + \mathbf{F}_N \tilde{\mathbf{w}}(i), \quad (3)$$

where $\tilde{\mathbf{H}}$ is the resulting channel matrix and $\tilde{\mathbf{w}}(i)$ is the $N \times 1$ truncated noise vector: $\tilde{\mathbf{w}}(i) := \mathbf{R}_{cp} \boldsymbol{\eta}(i)$, where $\mathbf{R}_{cp} :=$

$[\mathbf{0}_{N \times (P-N)}, \mathbf{I}_N]$ is the CP-removing matrix. Matrix $\tilde{\mathbf{H}}$ is an $N \times N$ circulant matrix with (k, l) th entry given by $h((k - l) \bmod N)$. Because (IFFTs diagonalize circulant matrices, the circulant matrix $\tilde{\mathbf{H}}$ can be decomposed as $\tilde{\mathbf{H}} = \mathbf{F}_N^H \mathbf{D}(\tilde{\mathbf{h}}) \mathbf{F}_N$, where $\tilde{\mathbf{h}} := [H(\exp(0)), H(\exp(j2\pi/N)), \dots, H(\exp(j2\pi(N-1)/N))]^T$ whose entries are the channel frequency response $H(z) := \sum_{l=0}^L h(l)z^{-l}$ evaluated at the subcarriers $z_k = \exp(j2\pi k/N)$, and $\mathbf{D}(\tilde{\mathbf{h}}) := \text{diag}(\tilde{\mathbf{h}})$ denotes a diagonal matrix with the (i, i) th entry being the i th element of the vector $\tilde{\mathbf{h}}$; see [9] for more details. Therefore, we can rewrite (3) as:

$$\mathbf{y}_{mc}(i) = \mathbf{D}(\tilde{\mathbf{h}})\mathbf{c}_{mc}s(i) + \mathbf{F}_N \bar{\mathbf{w}}(i). \quad (4)$$

With $\mathbf{D}(\mathbf{c}_{mc}) := \text{diag}(\mathbf{c}_{mc})$, we verify that $\mathbf{D}(\tilde{\mathbf{h}})\mathbf{c}_{mc} = \mathbf{D}(\mathbf{c}_{mc})\tilde{\mathbf{h}}$. Define $\mathbf{h} := [h(0), \dots, h(L)]^T$ and \mathbf{V} as the $N \times (L+1)$ Vandermonde matrix formed by the first $L+1$ columns of $\sqrt{N}\mathbf{F}_N$; thus, $\tilde{\mathbf{h}} = \mathbf{V}\mathbf{h}$ represents a scaled FFT operation in matrix form. We then can rewrite (4) as:

$$\mathbf{y}_{mc}(i) = \mathbf{D}(\mathbf{c}_{mc})\mathbf{V}\mathbf{h}s(i) + \mathbf{F}_N \bar{\mathbf{w}}(i). \quad (5)$$

Since the spreading sequence is binary, i.e., \mathbf{c}_{mc} has entries ± 1 , it holds that $\mathbf{D}^H(\mathbf{c}_{mc})\mathbf{D}(\mathbf{c}_{mc}) = \mathbf{I}_N$, and after multiplying (5) with $\mathbf{D}^H(\mathbf{c}_{mc})$ we arrive at

$$\mathbf{D}^H(\mathbf{c}_{mc})\mathbf{y}_{mc}(i) = \mathbf{V}\mathbf{h}s(i) + \mathbf{D}^H(\mathbf{c}_{mc})\mathbf{F}_N \bar{\mathbf{w}}(i). \quad (6)$$

Our primary goal is to compare the ability of MC-SS and DS-SS to combat multipath fading; therefore, we now describe the discrete time baseband model of DS-SS that is depicted in the lower part of Fig. 1.

Without FFT and CP insertion at the transmitter, the transmitted block in DS-SS is $\mathbf{u}_{ds}(i) = \mathbf{c}_{ds}s(i)$, where $\mathbf{c}_{ds} := [c_{ds}(0), c_{ds}(1), \dots, c_{ds}(P-1)]^T$ is a $P \times 1$ vector having the same block length as the MC-SS system (the upper part of Fig. 1). Replacing $\mathbf{u}(i)$ in (2) by $\mathbf{u}_{ds}(i)$, and with \mathbf{R}_{cp} eliminating IBI as in (3), we arrive at:

$$\mathbf{y}_{ds}(i) = \mathbf{R}_{cp}\mathbf{H}_0\mathbf{c}_{ds}s(i) + \bar{\mathbf{w}}(i). \quad (7)$$

Because $\mathbf{H}_0\mathbf{c}_{ds}$ represents in matrix-vector form the linear convolution between \mathbf{h} and \mathbf{c}_{ds} , we can commute \mathbf{h} and \mathbf{c}_{ds} to obtain $\mathbf{H}_0\mathbf{c}_{ds} = \mathbf{C}_{ds}\mathbf{h}$, with \mathbf{C}_{ds} denoting a $P \times (L+1)$ Toeplitz matrix with first column \mathbf{c}_{ds} and first row $[c_{ds}(0), 0, \dots, 0]$. Let us now define the truncated $N \times 1$ code vector for DS-SS as $\tilde{\mathbf{c}}_{ds} := \mathbf{R}_{cp}\mathbf{c}_{ds}$. Multiplying \mathbf{R}_{cp} with \mathbf{C}_{ds} yields a truncated $N \times (L+1)$ Toeplitz matrix $\tilde{\mathbf{C}}_{ds}$ with first column $\tilde{\mathbf{c}}_{ds}$ and first row $[c_{ds}(L), \dots, c_{ds}(0)]$. Therefore, we can rewrite (7) as:

$$\mathbf{y}_{ds}(i) = \tilde{\mathbf{C}}_{ds}\mathbf{h}s(i) + \bar{\mathbf{w}}(i). \quad (8)$$

Comparing (6) with (8), we *unify* MC-SS and DS-SS in the following equivalent model:

$$\mathbf{y}(i) = \mathbf{C}\mathbf{h}s(i) + \mathbf{w}(i) = \mathbf{c}s(i) + \mathbf{w}(i), \quad (9)$$

where $\mathbf{c} := \mathbf{C}\mathbf{h}$ denotes the equivalent signature code vector after channel convolution and receiver processing. For convenience, we list the corresponding vectors for MC-SS and DS-SS unified by (9):

$$\mathbf{c} = \mathbf{V}\mathbf{h}, \quad \mathbf{w}(i) = \mathbf{D}^H(\mathbf{c}_{mc})\mathbf{F}_N \bar{\mathbf{w}}(i), \quad \text{for MC-SS}, \quad (10)$$

$$\mathbf{c} = \tilde{\mathbf{C}}_{ds}\mathbf{h}, \quad \mathbf{w}(i) = \bar{\mathbf{w}}(i), \quad \text{for DS-SS}. \quad (11)$$

We assume the additive noise is white, i.e., $\mathbf{R}_{\bar{\mathbf{w}}\bar{\mathbf{w}}} := \mathbb{E}\{\mathbf{w}(i)\mathbf{w}^H(i)\} = \sigma_w^2\mathbf{I}_N$. Starting with the unifying model (9), the Maximum Ratio Combiner (MRC) output becomes: $\hat{s}(i) = \mathbf{c}^H\mathbf{y}(i)$. With $\sigma_s^2 := \mathbb{E}\{s(i)s^H(i)\}$, the output SNR becomes: $SNR = \mathbf{c}^H\mathbf{c}\sigma_s^2/\sigma_w^2$, where \mathbf{c} is defined in (10) for MC-SS and in (11) for DS-SS.

We next analyze the system bit error rate (BER) for random multipath channels.

3. RANDOM MULTIPATH FADING CHANNELS

Recall that $\mathbf{c} = \mathbf{V}\mathbf{h}$ for MC-SS and $\mathbf{c} = \tilde{\mathbf{C}}_{ds}\mathbf{h}$ for DS-SS. The corresponding SNRs for a given channel \mathbf{h} are:

$$SNR^{(mc)} = \mathbf{h}^H\mathbf{V}^H\mathbf{V}\mathbf{h}\sigma_s^2/\sigma_w^2 = N\mathbf{h}^H\mathbf{h}\sigma_s^2/\sigma_w^2, \quad (12)$$

$$SNR^{(ds)} = \mathbf{h}^H\tilde{\mathbf{C}}_{ds}^H\tilde{\mathbf{C}}_{ds}\mathbf{h}\sigma_s^2/\sigma_w^2. \quad (13)$$

Eqs. (12) and (13) clearly show that the SNR, and thus the BER, in MC-SS *do not* depend on the code choices, whereas they do so in DS-SS. In [4] it is assumed that the self-interference due to multipath is *negligible*, i.e., the shifts of the spreading code are nearly orthogonal to itself so that $\tilde{\mathbf{C}}_{ds}^H\tilde{\mathbf{C}}_{ds} = N\mathbf{I}_{L+1}$. Under this assumption, we have that $SNR^{(mc)} = SNR^{(ds)}$, which indicates that MC-SS and DS-SS exhibit the same ability in resisting multipath effects, which agrees with the results in [4]. In general, the Toeplitz matrix $\tilde{\mathbf{C}}_{ds}$ does not have orthogonal columns. The columns of $\tilde{\mathbf{C}}_{ds}$ can be approximately orthogonal (so that self-interference is negligible) only when the code length P is sufficiently large relative to the channel order L , and the code is well constructed. Unlike [4], where focus is placed on multiuser interference and narrow band interference but the multipath-induced self-interference is ignored, here, we explicitly consider this self-interference effect and compare the multipath resistance of DS-SS with that of MC-SS. Thanks to the FFT processing and the CP insertion at the transmitter, the FIR multipath is converted to parallel frequency-flat subchannels in MC-SS, so that the self-interference on each subcarrier is accounted for and absorbed in the fading coefficient for that subchannel. As confirmed by (12), the performance of MC-SS is independent of code choices. We next show the advantages of MC-SS over DS-SS in the randomly faded multipath channel scenario.

For random channels \mathbf{h} with covariance matrix $\mathbf{R}_{hh} := \mathbb{E}\{\mathbf{h}\mathbf{h}^H\}$, the BER for BPSK can be expressed in terms of the output SNR as: $P_b = \mathbb{E}_h\left\{\mathcal{Q}\left(\sqrt{SNR}\right)\right\}$. This expression is difficult to evaluate by averaging over the statistics of the fading amplitude random variables directly [8], since $\mathcal{Q}(x)$ is a nonlinear function of x . However, by using an alternative representation of $\mathcal{Q}(\cdot)$, a closed-form BER expression for independent faded channels has been obtained in [8]. Following the steps of [8], and assuming that the channel estimates at the receiver are error-free, we will first derive a general BER expression for MC-SS and DS-SS, and then compare their capabilities in resisting multipath.

We first diagonalize \mathbf{R}_{hh} via its spectral decomposition:

$$\mathbf{R}_{hh} = \mathbf{U}_h\mathbf{D}_h\mathbf{U}_h^H, \quad \mathbf{D}_h = \text{diag}(\lambda_{11}, \dots, \lambda_{LL}), \quad (14)$$

where \mathbf{U}_h is unitary and $\lambda_{ii} \geq 0$ denotes the i th eigenvalue of \mathbf{R}_{hh} . Similarly, we decompose the signature code covariance

matrix $\mathbf{R}_{cc} := \mathbb{E}\{\mathbf{c}\mathbf{c}^H\}$ as:

$$\mathbf{R}_{cc} = \mathbb{E}\{\mathbf{C}\mathbf{h}\mathbf{h}^H\mathbf{C}^H\} = \mathbf{C}\mathbf{R}_{hh}\mathbf{C}^H = \mathbf{U}_c\mathbf{D}_c\mathbf{U}_c^H, \quad (15)$$

where \mathbf{U}_c is a $P \times (L+1)$ matrix with orthonormal columns and \mathbf{D}_c is a diagonal matrix with entries $\bar{\lambda}_{ii}, i \in [1, L+1]$. When \mathbf{R}_{hh} is diagonal and \mathbf{C} has orthonormal columns, we have $\bar{\lambda}_{ii} = \lambda_{ii}, \forall i \in [1, L+1]$.

Pre-multiplying $\mathbf{y}(i)$ in (9) with \mathbf{U}_c^H yields:

$$\begin{aligned} \mathbf{y}'(i) &:= \mathbf{U}_c^H\mathbf{y}(i) = \mathbf{U}_c^H\mathbf{C}\mathbf{h}s(i) + \mathbf{U}_c^H\mathbf{w}(i) \\ &:= \mathbf{h}'s(i) + \mathbf{w}'(i), \end{aligned} \quad (16)$$

where $\mathbf{h}' := \mathbf{U}_c^H\mathbf{C}\mathbf{h}$ and $\mathbf{w}'(i) := \mathbf{U}_c^H\mathbf{w}(i)$ denote equivalent channel and noise vectors. Because $\mathbf{R}_{h'h'} = \mathbf{U}_c^H\mathbf{R}_{cc}\mathbf{U}_c = \mathbf{D}_c$, the entries of \mathbf{h}' are uncorrelated, while $\mathbf{w}'(i)$ is still white since $\mathbf{R}_{w'w'} = \sigma_w^2\mathbf{I}_{L+1}$. The MRC symbol estimate $\hat{s}(i) = (\mathbf{h}')^H\mathbf{y}'(i)$ equals the MMSE/MF receiver output operating on $\mathbf{y}(i)$: $\hat{s}(i) = \mathbf{c}^H\mathbf{y}(i)$. As a result, a closed form Symbol Error Rate (SER) expression for MPSK (M constellation points) signals can then be obtained by direct substitution from [8, eq. (44)]:

$$P_s(E) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \prod_{i=1}^L I_i(\bar{\lambda}_{ii}\sigma_s^2/\sigma_v^2, \gamma_{PSK}, \theta) d\theta, \quad (17)$$

where $\gamma_{PSK} := \sin^2(\pi/M)$, and $I_i(x, \gamma_{PSK}, \theta)$ is the moment of the probability density function of h'_i evaluated at $-\gamma_{PSK}/\sin^2(\theta)$ (see [8, eq. (24)]). For example, if h'_i is Rayleigh distributed, we have

$$I_i(x, \gamma_{PSK}, \theta) = [1 + \gamma_{PSK}x\sigma_s^2/(\sigma_v^2\sin^2(\theta))]^{-1}. \quad (18)$$

The moment $I_i(x, \gamma_{PSK}, \theta)$ for other distributions such as Nakagami, and the resulting SER for different constellations (e.g., QAM) can be found in [8].

To establish the optimality of MC-SS over DS-SS, let us consider the generic model of [3]:

$$\tilde{\mathbf{y}}(i) = \tilde{\mathbf{C}}\mathbf{h}s(i) + \tilde{\mathbf{w}}(i), \quad (19)$$

where $\tilde{\mathbf{w}}(i)$ is white and $\tilde{\mathbf{C}}$ is an arbitrary $N \times (L+1)$ matrix obeying the power constraint: $\text{tr}\{\tilde{\mathbf{C}}^H\tilde{\mathbf{C}}\} = \mathcal{P}_0$, prescribed by the transmit-power budget.

Starting with the generic model (19), it is possible to choose the precoder $\tilde{\mathbf{C}}$ according to the optimality criterion specified in the following theorem:

Theorem 1 [3]: *If \mathbf{h} and $\tilde{\mathbf{w}}(i)$ in (19) are uncorrelated and $\tilde{\mathbf{w}}(i)$ is white, the optimum precoding matrix $\tilde{\mathbf{C}}$ is given by: $\tilde{\mathbf{C}}_{opt} = \Phi\mathbf{D}_f\mathbf{U}_h^H$, where \mathbf{U}_h is defined in (14); diagonal matrix \mathbf{D}_f is the optimal power loading matrix selected as in [3, eq. (17) and (18)], and Φ an arbitrary $N \times (L+1)$ matrix with orthonormal columns. Optimality of $\tilde{\mathbf{C}}_{opt}$ pertains to either minimizing the error in estimating the random channel, $E\{\|\mathbf{h} - \hat{\mathbf{h}}\|^2\}$, or, maximizing the conditional mutual information $I(\bar{\mathbf{x}}, \mathbf{h}|s)$ if \mathbf{h} is complex Gaussian distributed.*

If the entries of \mathbf{h} are independent and identically distributed (i.i.d.), i.e., $\mathbf{R}_{hh} = \sigma_h^2\mathbf{I}$ with $\mathbf{U}_h = \mathbf{I}_{L+1}$, then the

optimal power loading matrix $\mathbf{D}_f = \alpha\mathbf{I}_{L+1}$, where $\alpha^2 = \mathcal{P}_0/(L+1)$ [3]. In this case, the optimal precoder $\tilde{\mathbf{C}}_{opt} = \alpha\Phi$ should have orthogonal columns. Because the Vandermonde matrix \mathbf{V} has orthogonal columns while the Toeplitz matrix $\tilde{\mathbf{C}}_{ds}$ does not, MC-SS is optimal in this setting and it thus outperforms DS-SS considerably.

The optimality in Theorem 1 amounts to minimizing the mean-square channel estimation error, which implies that channel estimation accuracy dictates the overall BER performance. However, for special cases, it is possible to have the power loading of Theorem 1 optimize the overall BER directly (see, e.g., [2] for differential QPSK constellations which lead to a simple closed-form BER expression).

However, when the entries of \mathbf{h} are i.i.d. with Gaussian distribution and covariance matrix $\mathbf{R}_{hh} = \sigma_h^2\mathbf{I}$, we can directly establish the optimality based on the SER expression in (17). Because $\mathbf{R}_{hh} = \sigma_h^2\mathbf{I}_{L+1}$, we have \mathbf{D}_c in (15) for MC-SS as: $\mathbf{D}_c^{(mc)} = N\sigma_h^2\mathbf{I}_{L+1}$. Therefore, $\mathbf{D}_c^{(mc)}$ for MC-SS has equal diagonal entries, which is not the case for DS-SS because $\tilde{\mathbf{C}}_{ds}$ for DS-SS in (15) does not have orthogonal columns in general. However, the total transmitted power is the same because

$$\text{tr}\{\mathbf{D}_c^{(ds)}\} = \sigma_h^2\text{tr}\{\tilde{\mathbf{C}}_{ds}^H\tilde{\mathbf{C}}_{ds}\} = N(L+1)\sigma_h^2 = \text{tr}\{\mathbf{D}_c^{(mc)}\}.$$

Let us denote the i th diagonal element of $\mathbf{D}_c^{(ds)}$ by $\bar{\lambda}_{ii}^{(ds)}$ and of $\mathbf{D}_c^{(mc)}$ by $\bar{\lambda}_{ii}^{(mc)}$. We then have $\bar{\lambda}_{ii}^{(mc)} = (\sum_{i=1}^{L+1}\bar{\lambda}_{ii}^{(ds)})/(L+1)$. Applying the inequality: $(x_1 + x_2 + \dots + x_N) \geq N(x_1x_2 \dots x_N)^{1/N}, x_i > 0$, we obtain $(x_1x_2 \dots x_N)^{-1} \geq [(x_1 + x_2 + \dots + x_N)/N]^{-N} \geq 0$, and after taking into account (18), we arrive at the following inequality:

$$\begin{aligned} &\prod_{i=1}^{L+1} I_i(\bar{\lambda}_{ii}^{(ds)}\sigma_s^2/\sigma_v^2, \gamma_{PSK}, \theta) \\ &= \prod_{i=1}^{L+1} \left[1 + \frac{\gamma_{PSK}\bar{\lambda}_{ii}^{(ds)}\sigma_s^2}{\sigma_v^2\sin^2(\theta)}\right]^{-1} \geq \left[I_i(\bar{\lambda}_{ii}^{(mc)}\frac{\sigma_s^2}{\sigma_v^2}, \gamma_{PSK}, \theta)\right]^{L+1} \end{aligned} \quad (20)$$

Substituting (20) back into (17), we thus obtain:

$$P_s^{(ds)}(E) \geq P_s^{(mc)}(E), \quad (21)$$

where equality is achieved when the Toeplitz matrix $\tilde{\mathbf{C}}_{ds}$ for DS-SS has orthogonal columns, i.e., when self-interference is zero. Inequality (20) implies that equal power loading optimizes BER for i.i.d. Gaussian channels. By distributing its power evenly across all subbands, MC-SS provides maximum protection against random frequency-selective multipath fading in this case.

If \mathbf{h} is not i.i.d., equipower loading $\mathbf{D}_f = \alpha\mathbf{I}_{L+1}$ turns out to be near optimal at high SNR [3]. The selected precoder matrix $\tilde{\mathbf{C}} = \alpha\Phi\mathbf{U}_h$ has orthogonal columns, which corroborates the near-optimality of MC-SS at high SNR.

To shed further light on the performance of digital MC-SS relative to DS-SS and to study the code dependence of DS-SS, we consider the following scenarios.

We construct three channel models, assuming that the channel \mathbf{h} is Gaussian distributed of order $L = 2$, Channel 1 is i.i.d. with $\mathbf{R}_{hh} = \text{diag}(1, 1, 1)/3$; channel 2 has $\mathbf{R}_{hh} = \text{diag}(1, 0.5, 0.1)/1.6$, i.e., the first path shows a 3dB gain over

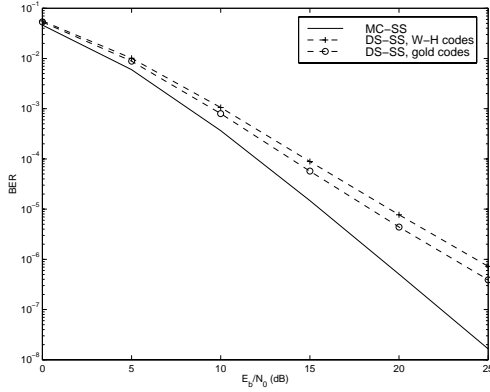


Fig. 2. MC-SS versus DS-SS with $P = 8$

the second and 10dB gain over the third path; and channel 3 is adopted from [2] with $\mathbf{R}_{hh} = \text{diag}(1, 0.05, 0.01)/1.06$, i.e., the first path has a 13dB gain over the second and 20dB gain over the third path.

To avoid the code dependence for DS-SS, we adopt the code-hopping scheme of [5] and average the BER over all possible code choices. It is known that W-H codes have poor autocorrelation properties. Therefore, we also employ Gold codes, which have better autocorrelation properties [6]. In Figs. 2 and 3 we compare the BER of MC-SS with the average BER of DS-SS with W-H codes of length $P = 8, 16$ and with Gold codes $P = 7, 15$, respectively. First, we see that MC-SS outperforms DS-SS with W-H codes considerably because the multipath induced self-interference of W-H codes is large. When Gold sequences are employed, we observe that the BER of DS-SS approaches that of MC-SS when the code length increases, as the self-interference becomes relatively smaller and smaller. In Fig. 2, note that MC-SS offers a 4 to 5 dB advantage over DS-SS at BER of 10^{-6} .

With colored channels, we observe similar results as those in Figs. 2 and 3 for i.i.d channels. We compare in Fig. 4 MC-SS against DS-SS with code length 16 for both channels 2 and 3. Although MC-SS is not optimum (near optimum at high SNR) in these two channel settings, we clearly see that MC-SS outperforms DS-SS alternatives considerably, especially when the spreading codes for DS-SS are not well constructed. In a nutshell, the superiority of MC-SS over DS-SS in the presence of multipath justifies its increasing popularity.

4. CONCLUSIONS

We used results from [3] for the optimal coding matrix, and showed that in the case of uncorrelated and equal power paths, the optimal code leads to multi-carrier spread-spectrum (MC-SS) which may significantly outperform direct-sequence spread spectrum (DS-SS). We developed closed-form expressions for the BER performance of digital MC-SS and DS-SS schemes in the presence of frequency-selective multipaths (which destroy code orthogonality). The performance of MC-SS does not depend upon the spreading code; in contrast, the performance of DS-SS does depend upon the spreading code. In general, MC-SS outperforms DS-SS; the performance of DS-SS approaches that of MC-SS if the spreading gain is large and the codes are well chosen. In the case of colored channels (correlated paths and/or paths with unequal powers), MC-SS outperforms DS-SS, especially for short spreading lengths.

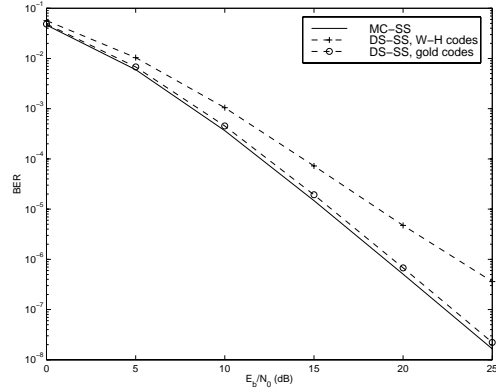


Fig. 3. MC-SS versus DS-SS with $P = 16$

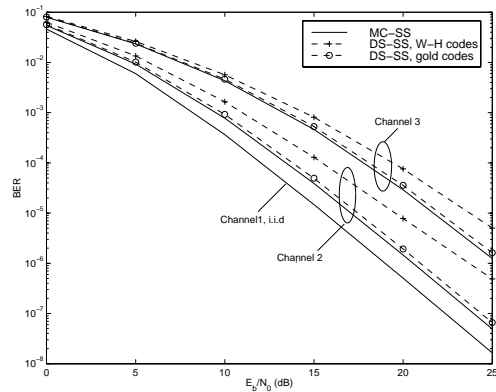


Fig. 4. MC-SS vs DS-SS, different channels

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